

# 5-1 Study Guide and Intervention

## Operations with Polynomials

**Multiply and Divide Monomials** Negative exponents are a way of expressing the multiplicative inverse of a number.

|                           |  |
|---------------------------|--|
| <b>Negative Exponents</b> | $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer $n$ . |
|---------------------------|--|

When you **simplify an expression**, you rewrite it without powers of powers, parentheses, or negative exponents. Each base appears only once, and all fractions are in simplest form. The following properties are useful when simplifying expressions.

|                             |   |
|-----------------------------|---|
| <b>Product of Powers</b>    | $a^m \cdot a^n = a^{m+n}$ for any real number $a$ and integers $m$ and $n$ .  |
| <b>Quotient of Powers</b>   | $\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers $m$ and $n$ .   |
| <b>Properties of Powers</b> | For $a, b$ real numbers and $m, n$ integers:<br>$(a^m)^n = a^{mn}$<br>$(ab)^m = a^m b^m$<br>$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$<br>$\left(\frac{a}{b}\right)^{-n} = \left(\frac{a}{b}\right)^n$ or $\frac{b^n}{a^n}, a \neq 0, b \neq 0$ |

**Example: Simplify. Assume that no variable equals 0.**

a.  $(3m^4n^{-2})(-5mn)^2$

$$\begin{aligned} (3m^4n^{-2})(-5mn)^2 &= 3m^4n^{-2} \cdot 25m^2n^2 \\ &= 75m^4m^2n^{-2}n^2 \\ &= 75m^{4+2}n^{-2+2} \\ &= 75m^6 \end{aligned}$$

b.  $\frac{(-m^4)^3}{(2m^2)^{-2}}$

$$\begin{aligned} \frac{(-m^4)^3}{(2m^2)^{-2}} &= \frac{-m^{12}}{\frac{1}{4m^4}} \\ &= -m^{12} \cdot 4m^4 \\ &= -4m^{16} \end{aligned}$$

**Exercises:**

**Simplify. Assume that no variable equals 0.**

1.  $c^{12} \cdot c^{-4} \cdot c^6$

2.  $\frac{b^8}{b^2}$

3.  $(a^4)^5$

4.  $\frac{x^{-2}y}{x^4y^{-1}}$

5.  $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$

6.  $\left(\frac{x^2y}{xy^3}\right)^2$

7.  $\frac{1}{2}(-5a^2b^3)^2(abc)^2$

8.  $m^7 \cdot m^8$

9.  $\frac{8m^3n^2}{4mn^3}$

10.  $\frac{2^3c^4t^2}{2^2c^4t^2}$

11.  $4j(-j^{-2}k^2)(3j^3k^{-7})$

12.  $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$

## Operations with Polynomials

|                   |  |
|-------------------|--|
| <b>Polynomial</b> | a monomial or a sum of monomials                                 |
| <b>Like Terms</b> | terms that have the same variable(s) raised to the same power(s) |

To add or subtract polynomials, perform the indicated operations and combine like terms.

**Example 1: Simplify**  $4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$ .

$$4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$$

$$= 4xy^2 + 12xy - 7x^2y - 20xy - 5xy^2 + 8x^2y$$

Distribute the minus sign.

$$= (-7x^2y + 8x^2y) + (4xy^2 - 5xy^2) + (12xy - 20xy)$$

Group like terms.

$$= x^2y - xy^2 - 8xy$$

Combine like terms.

### Exercises:

**Simplify.**

1.  $(6x^2 - 3x + 2) - (4x^2 + x - 3)$

2.  $(7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)$

3.  $(-4m^2) - 6m - (6m + 4m^2)$

4.  $27x^2 - 5y^2 + 12y^2 - 14x^2$

5.  $\frac{1}{4}x^2 - \frac{3}{8}xy + \frac{1}{2}y^2 - \frac{1}{2}xy + \frac{1}{4}y^2 - \frac{3}{8}x^2$

6.  $24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p$

**Find each product.**

7.  $2x(3x^2 - 5)$

8.  $7a(6 - 2a - a^2)$

9.  $(x^2 - 2)(x^2 - 5)$

10.  $(x + 1)(2x^2 - 3x + 1)$

11.  $(2n^2 - 3)(n^2 + 5n - 1)$

12.  $(x - 1)(x^2 - 3x + 4)$